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Inverse Filter Techniques in Digital Nuclear Signal Processing

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Summary

Introduction

Digital signal processing has entered all areas of nuclear signal analysis. Deconvolution or inverse filters are the base for improved energy resolution, fast timing and high throughput nuclear electronics. Deconvolution filtering allows processing scintillation detectors with complex decay schemes achieving unparalleled high throughput for detectors with long decay times. The light decay time is no longer a limiting factor for system throughput.

Signals from radiation detectors are subject to transfer functions of the detector physics and the front-end electronics before being measured. Especially energy and timing information are affected. If the analogue transfer function can be modeled by an LTI (Linear Time Independent) system, then an inverse filter, called deconvolution, reveals the original unchanged input signal. We will present a method for the construction of general linear deconvolution filters for scintillation as well as semiconductor spectrometry. The differential equation characterizing the transfer network, detector and electronics, is translated to a difference equation after signal sampling and digitization. We will show that discrete LTI systems can be inverted, and how to construct inverse difference equations.

With the first commercially available digital nuclear signal processing system called PPADC introduced in 1994, the problem of the exponential signal deconvolution and digital trapezoidal filtering had already been resolved [1] [2]. A procedure called MWD (Moving Window Deconvolution) was implemented on cascaded floating point DSPs accomplishing digital multi-pole-zero cancellation and ballistic deficit correction. This paper will generalize the MWD method introducing inverse LTI filtering for the deconvolution of all nuclear electronic signals. These filters are represented by the well known FIR (Finite Impuls Response) and IIR (Infinite Impulse Response) filters which, although implemented as deconvolution in the time domain, are typically foremost studied in the frequency domain.

Because of the nuclear signal characteristics, a direct treatment in the time domain leads to very useful new results. Still we benefit a lot from those frequency filter construction methods that search to transfer analogue designs into the digital domain.

There are two important features applying to nuclear signals that allow a straight forward deconvolution approach in the time domain. For one, the signals can be understood as generated by random events of short, finite duration. Secondly, the signals are transferred by an analogue linear network. From linear filter theory we know, that the impulse response is the system function of the network which we are studying. Thus we not only measure the original event but we even principally know the complete transfer network from a single pulse.

Mathematical Recapitulation

Linear Time-Invariant Filtering [3]

Signal processing operations such as signal transmission are implemented with linear time-invariant operators. The time invariance of an operator L means that if the input $f(t)$ is delayed by τ , $f_\tau(t) = f(t-\tau)$, then the output is also delayed by τ: $g(t) = Lf(t) \Rightarrow g(t-\tau) = Lf_\tau(t)$.

Impulse Response

Linear time-invariant systems (filters, transfer functions) are characterized by their response to a Dirac impulse. If f is continuous, its value at t can be formally written as an integration against a Dirac located at t. Let $\delta_u(t) = \delta(t-u)$:

$$f(t) = \int_{-\infty}^{+\infty} f(u)\delta_u(t)du \Rightarrow Lf(t) = \int_{-\infty}^{+\infty} f(u)L\delta_u(t)du$$

Let h be the impulse response of L: $h(t) = L\delta(t)$. The time invariance proves that $L\delta_u(t) = h(t-u)$ and hence

$$Lf(t) = \int_{-\infty}^{+\infty} f(u)h(t-u)du = h * f(t)\cdot$$

A time-invariant filter is thus equivalent to a convolution of f with the impulse response h.

Radiation Detector Response Function as a Linear Filter

We assume a scintillation detector system. The basic differential equation for the PMT current output fed to an RC high pass filter is:

$$I_{pmt}(t) = \frac{1}{RC}\dot{I}_{pmt}(t)$$

Excited by Gamma quanta, ideal scintillators emit light pulses with a single decay component. An equivalent differential equation is:

$$l(t) = \frac{1}{\tau}\dot{l}(t)$$

The time domain output signal of the detector-PMT combination is the convolution

$$I_t(t) = \int_{-\infty}^{\infty} I_{pmt}(u)\cdot l(t-u)du$$

which becomes a multiplication in the s-domain after a Laplace transform:

$$I_s(s) = \frac{1}{(s-\tau)\cdot(s-RC)}$$

Formally, the *detectors physics* (light emission process) can thus be descibed – and handled – in the same manner as the *electronics* transfer network shaping the detector signal. In general, *any* system characterized by a linear differential equation can be treated as linear time invariant filter, thereby allowing a variety of solutions and methods of equivalent filter problems. A simple model for a general detector-preamplifier system is an all-pole filter [4]. In the typical s-transform (Laplace) notation the general all pole transfer function is

$$H(s) = \frac{1}{\prod_{\nu=1}^{N}(s-\sigma_\nu)} \qquad (1)$$

The detector-preamplifier pulse output then is modeled as the impulse response of an all-pole filter. As a linear filter is completely characterized by its impulse response to a Dirac, each detector pulse carries the complete information of the transfer network *which includes the detector physics,* e.g the light emission process in the scintillator.

Inverse Filtering or Deconvolution

The goal is measuring the properties of the "original" initial detector impulse representing the energy (or charge) of the event, the number of electron-hole pairs generated in the active detector volume. The proposed solution is applying a filter inverting the differential equation which is characterizing the *whole* detector system including the electronics.

Such linear deconvolution is understood as multiple pole-zero cancellation process, whereby all poles characterizing the analogue electronics and the detector are cancelled by corresponding zeroes of the inverse filter:

$$H^*(s) = \prod_{\nu=1}^{N}(s-\lambda_\nu) \qquad (2).$$

Setting $\sigma_\nu = \lambda_\nu$ in (1) it follows that $H(s)\cdot H^*(s) \equiv 1$.

The continuous inverse filter (2) cannot be practically realized utilizing discrete analogue electronics. However, in digital signal processing the appropriate algorithms can be applied:

If an analogue LTI process can be modeled in the digital domain as discrete LTI filter with sufficient accuracy, it is possible to construct the inverse filter revealing the original input signal.

The General LTI Difference Equation and its Inverse

In the digital domain the general difference equation for a recursive discrete LTI (DLTI) filter defines the sequence y_n as mapping of the sequence x_n; it defines a subclass of all LTI filters:

$$y_n = \sum_{\nu=0}^{N} c_\nu x_{n-\nu} + \sum_{\nu=1}^{M} d_\nu y_{n-\nu}$$

The DLTI equation states how to calculate the filter output y_n from the input x_n. The base of this paper is the answer to the question: *Is the DLTI filter invertible?* [5]. Since the mapping of x_n on y_n is bijective the answer is obvious:

$$H(z) = \frac{\sum_{\nu=0}^{N} c_\nu z^{-\nu}}{1-\sum_{\nu=1}^{M} d_\nu z^{-\nu}} \quad \Leftrightarrow \quad H^{-1}(z) = \frac{\sum_{\nu=0}^{N} d'_\nu z^{-\nu}}{1-\sum_{\nu=1}^{M} c'_\nu z^{-\nu}}$$

We will exploit the inversion property [6] of LTI filters following the path:

1. A detector system is uniquely characterized by a linear differential equation $L(t)$.
2. The differential equation is interpreted as an analogue linear filter $H(s)$.
3. The analogue filter is digitized and translated into an equivalent digital filter $\mathrm{H}(z)$.
4. The digital filter is inverted: $\mathrm{H}^{-1}(z)$.
5. The inverse filter applied to the sampled detector signal reveals the input pulse $h(t_n)$:

$$L(t) \rightarrow H(s) \rightarrow \mathrm{H}(z) \rightarrow \mathrm{H}^{-1}(z) \rightarrow h(t_n).$$

Applications

As a direct consequence of the inverse filter construction we are able to present solutions for key applications utilizing such deconvolution filtering:

1. Deconvolution signal processing of a standard NaI(Tl) scintillator;
2. Deconvolution of $CdWO_4$ signals as an example for deconvolution of two decay components;
3. Pole zero cancellation for high resolution digital spectrometers and pole-zero-free systems.

The technique of inverse filtering is well suited for improving the performance of digital gamma spectroscopy systems. Practical applications will be discussed.

References

[1] J.Stein, A.Georgiev, A.Büchner and W.Gast, US Patent No. 5 307 299 (1994).

[2] J.Stein, F.Scheuer, W.Gast, and A. Georgiev, "X-ray detectors with digitized preamplifiers", Nucl. Instr. Meth. B113 (1996), pp 141-145; in: Kubik, Döbeli, Suter (Editors), Accelerators in applied research and technology, ECAART-4, 1995.

[3] Mallat, Wavelet Tour of Signal Processing, Academic Press, 1999, p. 20ff.

[4] Arthur B. Williams, Fred J. Taylor, Electronic Filter Design Handbook, McGraw-Hill 1988.

[5] John G. Proakis, Dimitris G. Monolakis, 3rd Edition Digital Signal Processing, Principles, Algorithms, and Applications, Prentice–Hall, 1996.

[6] Karl J. Åström, Björn Wittenmark, Computer Controlled Systems, Prentice-Hall, 1997